Example of Solving **Dimensional Analysis**

Question

Can you work out the dimensions for this proportionality and then work out the equation from the values you worked out

$$(s) \propto (u) * (t) + (a) * (t)$$

Where:

- s = Displacement
- u = initial velocity
- t = time
- a = acceleration

Experiment results:

- Initial velocity, $u = 0$
- Acceleration, $a = 2$
- Time, $t = 4$
- Measured displacement: $s = 16$

Step 1

Work out each component's fundamental format

- Displacement (s) = [L]
- Initial velocity (u) = ?
- Time (t) = [T]
- Acceleration (a) = ?

Working out initial velocity:

- Velocity = $\frac{\text{Distance}}{\text{Time}}$
- Distance = [L]
- Time = [T]
- Velocity = $\frac{[L]}{[T]}$

Working out acceleration:

- Acceleration = $\frac{\Delta v}{\Delta t}$
- Velocity = $\frac{[L]}{[T]}$
- Acceleration = $\frac{[L]}{[T^2]}$

Step 2

Put the fundamentals back into the equation and then apply the negative exponent rule to bring terms up from the denominator

$$[L] \propto \frac{[L]}{[T]} * [T] + \frac{[L]}{[T^2]} * [T]$$

$$[L] \propto [L * T^{-1}] * [T] + [L * T^{-2}] * [T]$$

Step 3

Split the fundamental format at the plus then add in our (a,b,c,d)

$$[L] \propto [L * T^{-1}] * [T]$$

$$[L] \propto [L * T^{-2}] * [T]$$

$$[L^1] \propto [L^a * T^{-1a}] * [T^b]$$

$$[L^1] \propto [L^c * T^{-2c}] * [T^d]$$

Step 4

Combine like terms

$$[L^1] \propto [L^a * T^{-a+b}]$$

$$[L^1] \propto [L^c * T^{-2c+d}]$$

Step 5

Work out the values for (a,b,c,d)

$$[L^1 T^0] \propto [L^a * T^{-a+b}]$$

$$a = 1$$

$$-a + b = 0$$

$$-1 + b = 0$$

$$b = 1$$

$$[L^1 T^0] \propto [L^c * T^{-2c+d}]$$

$$c = 1$$

$$-2c + d = 0$$

$$-2 + d = 0$$

$$d = 2$$

Step 6

Put the (a,b,c,d) values back into the equation with the constants k_1 and k_2

$$s = k_1(u^1 * t^1) + k_2(a^1 * t^2)$$

$$s = k_1(u * t) + k_2(at^2)$$

Step 7

We can use our science logic to get rid of one of our constants here, we know that if something has 0 acceleration then we must have uniform motion.

This means we could just do $\boldsymbol{s = ut}$ which can also be written as $\boldsymbol{s = 1(ut)}$

So for this all to be true k_1 must be 1

$$s = 1(u * t) + k_2(at^2)$$

$$s = u * t + k_2(at^2)$$

Step 8

We can now finally put in our values from our “experiment” to find the value for k_2

$$16 = 0 * 4 + k_2(2 * 4^2)$$

$$16 = k_2(2 * 4^2)$$

$$16 = k_2(32)$$

$$k_2 = \frac{1}{2}$$